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Cut-off frequency prediction for MMW coaxial interconnects

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Abstract: Due to new millimeter wave (MMW) bands being used for the 5G communication effort, hardware designers will need to carefully consider high frequency cables and interconnects, with respect to minimizing losses through their maximum operating frequency. This maximum frequency, or cut-off frequency, is considered to be when higher order modes (such as the TE_{11} mode) can propagate, robbing signal from the fundamental TEM mode. This frequency is determined by the transmission line and connectors in the cable assembly and can be difficult to accurately predict. Understanding the effect of supports beads and impedance matching can help provide a model of this phenomenon. Simulating the connector and cable using ANSYS® HFSS™ software will validate the model and provide a more accurate prediction of the cut-off frequency for the assembly. This paper will present a model of the phenomenon and simulation to support the model in order to better predict the cut-off frequency of a coaxial cable assembly.

Main Body:

Dielectric support beads used in coaxial connectors serve multiple functions. They position the center pin concentric to the outer conductor and keep the pin from moving in or out along the axis of connector. The bead must also be impedance matched and, through the use of high-impedance compensating features, account for capacitance due to changes in conductor diameters. One of the downsides of support beads is that they often decrease the effective TE_{11} mode, or “cut-off” frequency of the assembly. The cut-off frequency signifies the maximum operating frequency for single, TEM mode propagation. The best way to determine the effective cut-off frequency of a connector design is to simulate it in a full-field simulation package, like ANSYS® HFSS™ software.

Figure 1 a) shows a field plot of the TEM mode, shown on the bead face. You can see that the fields emanate radially and are axially symmetric. Figure 1 b) shows the fields of the TE_{11} mode on the face of the bead. You can see that, when this mode is present, “lobes” exists on either side of the center conductor. You can create this effect in your connector simulation by introducing a small asymmetry in the coax cross section (like moving the center conductor and bead off center a bit).

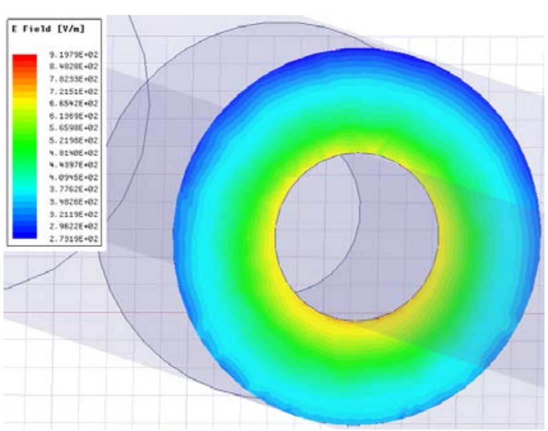


Figure 1a) TEM mode (off resonance)

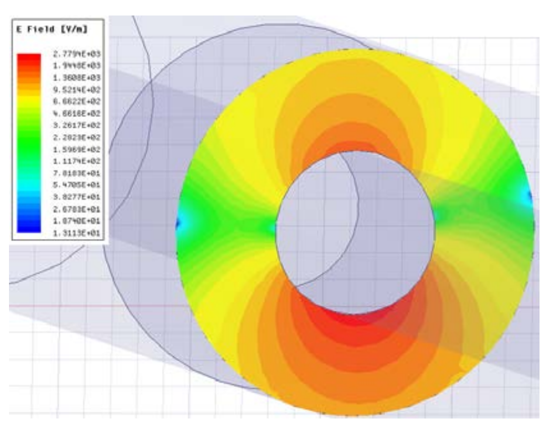


Figure 1b) TE11 mode (at resonance)

Another way to examine the cut-off frequency of a bead is by using a technique described in a paper by Gilmore[1]. He describes a process where you first calculate the frequency of the TE_{11} mode for the airline section on either side of the bead and for the main section in the middle of the bead, as if they were infinitely long. There is a simple approximation for the cut-off frequency of a coaxial section shown in Equation #1.

Equation #1: $f_c \approx \dfrac{190.85}{(d+D)\sqrt{\varepsilon_r}}$ (in GHz)

; where d and D are inner and outer diameters (in mm)
ε_r is the effective dielectric constant of the section

However, this approximation can be off by several hundred MHz. It is best to carry out the full solution (described by Dimitrios[2]) and solve the "characteristic Bessel equation" *J'n(X)*Y'n(kX)-J'n(kX)*Y'n(X)=0* for each section. This will give an accurate value for the TE_{11} mode frequency. The next step is to calculate the input impedance (from the perspective of the TE wave, not the TEM wave) at each interface of the bead. Figure #2 shows an illustration of the single support bead.

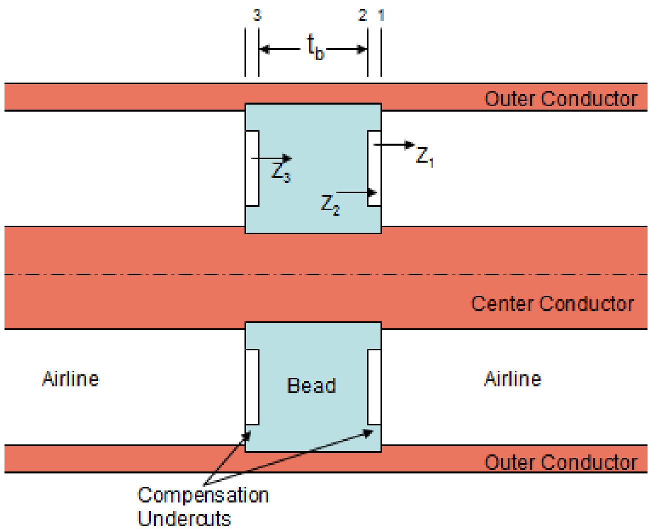


Figure #2: single bead illustration with the impedance labels for each interface

The first assumption is that the compensation sections on either side of the bead can be included into the airline section (since it should be impedance matched). This means that the length of the bead section equals t_b and that Z_2 equals Z_1. The next assumption is that the length of the airline section is sufficiently long enough that Z_1 equals the input impedance of the airline (Z_{0a}) such that;

$$Z_2 = Z_1 = Z_{0a} = j\eta_0 \frac{1}{\sqrt{1+\left(\frac{f}{f_{ca}}\right)^2}}$$

where; $\eta_0 = 376.7$ ohms, f_{ca} = TE_{11} frequency of the infinite airline section

$f =$ frequency of interest (below f_{ca})

You can see that the TE wave impedance in the airline section is purely imaginary below f_{ca}. This is your first design requirement; the cut-off frequency for the airline section must be higher than the desired maximum operating frequency for the connector.

The final step will be to examine the input impedance at the left face of the bead (Z_3). Using transmission line theory, you can calculate Z_3 in terms of the terminating impedance of the airline (Z_{0a}), the TE wave impedance within the bead (Z_{0b}), and the propagation constant within the bead (γ_b).

$$Z_3 = Z_{0b} \frac{Z_{0a} + Z_{ob} \tanh(\gamma_b t_b)}{Z_{0b} + Z_{oa} \tanh(\gamma_b t_b)}$$

where; $\gamma_b =$ Propagation constant within the bead

$t_b =$ Length of bead section

Since the cut-off frequency of the bead section (f_{cb}) is most likely lower than the airline section, we are interested in the frequencies between f_{cb} and f_{ca} and where that actual TE_{11} mode frequency will end up. With this assumption, we can calculate Z_{0b} and γ_b.

$$Z_{0b} = \eta_b \frac{1}{\sqrt{1-\left(\frac{f_{cb}}{f}\right)^2}} \quad \text{and} \quad \gamma_b = jk_b\sqrt{1-\left(\frac{f_{cb}}{f}\right)^2}$$

where; $\eta_b = \dfrac{376.7}{\sqrt{\varepsilon_{rb}}}$ Ohms ($\varepsilon_{rb} =$ dielectric constant in bead)

$k_b = 2\pi\sqrt{\varepsilon_{rb}}\dfrac{f}{c_0}$ ($c_0 =$ speed of light in vacuum)

$f_{cb} = TE_{11}$ frequency of the bead section as if it were infinitely long

You can use the identity *tanh(jx) = jtan(x)* to simplify the expression for Z_3. Gilmore[1] states that the frequency, or frequencies, at which Z_3 and Z_{0a} are complex conjugates of each other will support a TE_{11} mode resonance. The first surprising result is that there are conditions where the TE_{11} mode can resonant at multiple frequencies.

To show this in action, we have created a simple experiment where we take a 10cm long, 7mm diameter airline and inserted a solid PTFE bead (0.121 inch ID, 0.273 inch OD, lengths from 0.125 inches to 2 inches) shown in Figure #3. When the bead sits in the airline, there is just enough non-concentricity to excite the TE_{11} modes.

Figure #3: Photo of MMC 2653D 7mm airline and PTFE beads

Table 1 shows the TExx modes for the infinitely long 7mm airline and the infinitely long bead sections. You can see that the TE_{11} mode is the lowest in frequency and the next higher mode does not occur until 27 GHz. Therefore, any resonances below 20 GHz must be due to the TE_{11} mode.

Table#1: TExx modes for PTFE bead section and 7mm airline section

Mode	Bead	T-Line
TE11	13.8	19.4
TE21	27.0	38.0
TE31	39.4	55.4
TM01	53.3	75.0
TE01	55.1	77.6
TM11	55.1	77.6
TE12	57.4	80.7
TM21	60.2	84.6
TE22	63.7	89.6

The following figures #4 through #8 show results for the 0.125, 0.250, 0.500, 1.0, and 2.0 inch long PTFE beads respectfully. The first plot (a) is the output from the Gilmore[1] calculation. The (b) plot has the modeled data using ANSYS® HFSS™ software (orange) and the measured Insertion Loss data [$20Log_{10}(|S_{21}|)$] taken from the 7mm airline and bead using a vector network analyzer (blue).

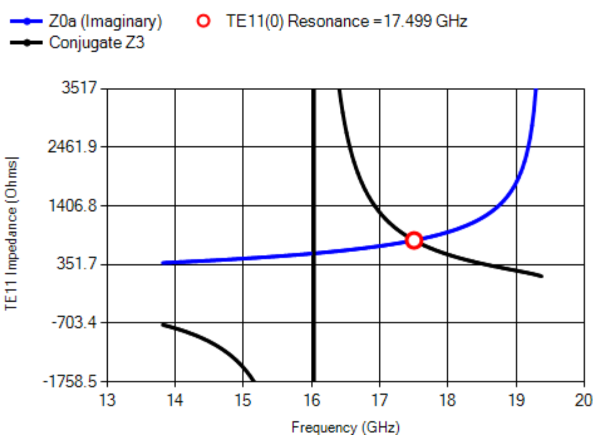


Figure #4a: Z_{0a} and Z_3 for 0.125" bead

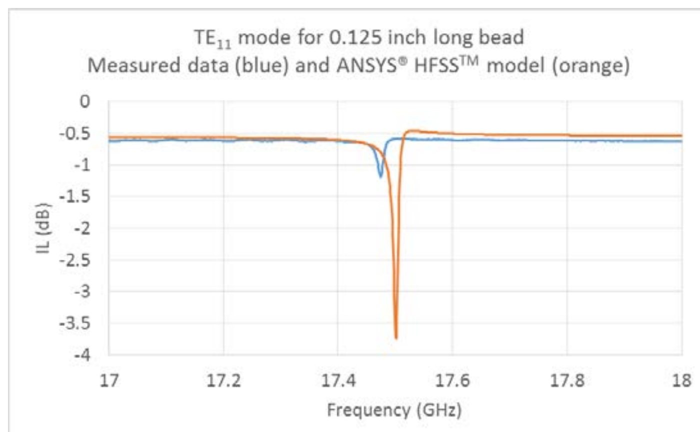


Figure #4b: Model and measured data for 0.125" bead

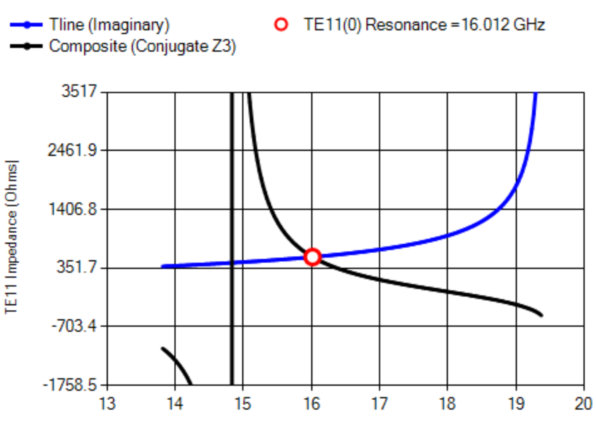


Figure #5a: Z_{0a} and Z_3 for 0.250" bead

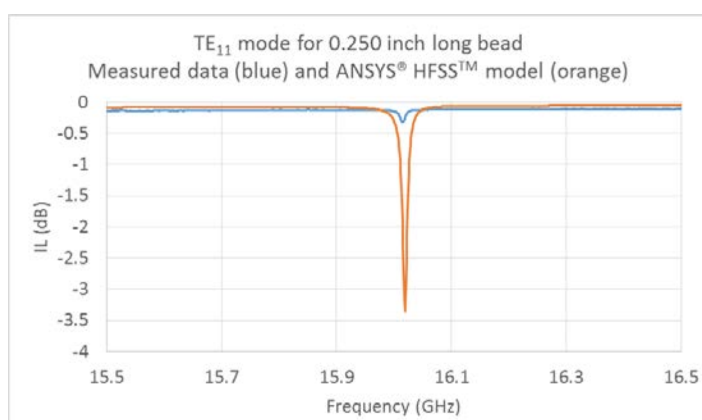
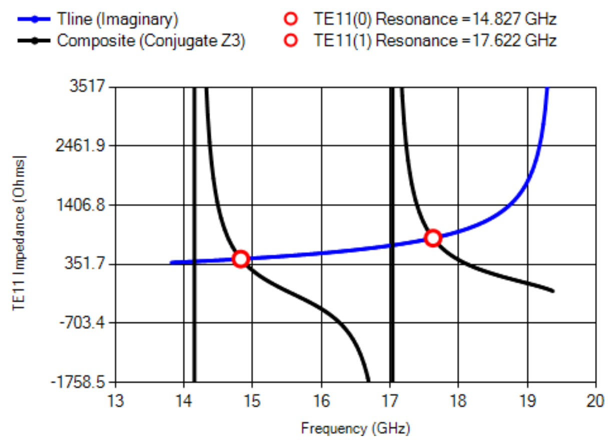


Figure #5b: Model and measured data for 0.250" bead



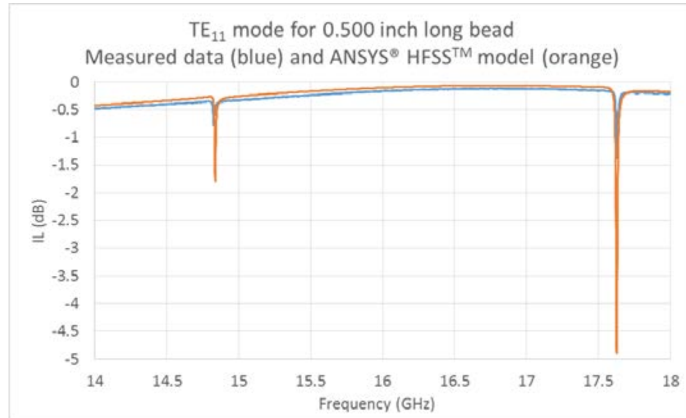
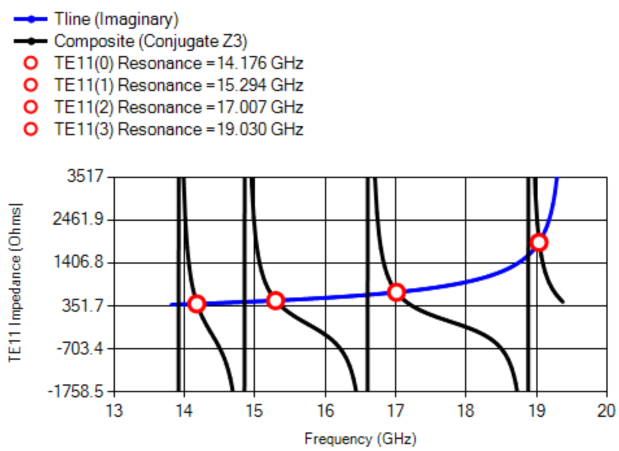


Figure #6a: Z_{0a} and Z_3 for 0.500" bead

Figure #6b: Model and measured data for 0.500" bead



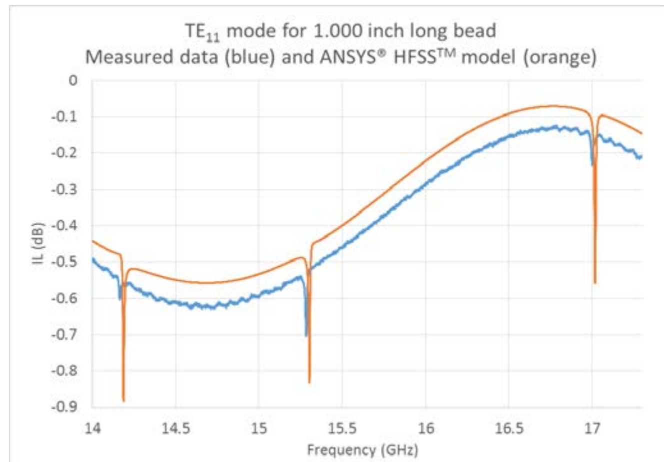
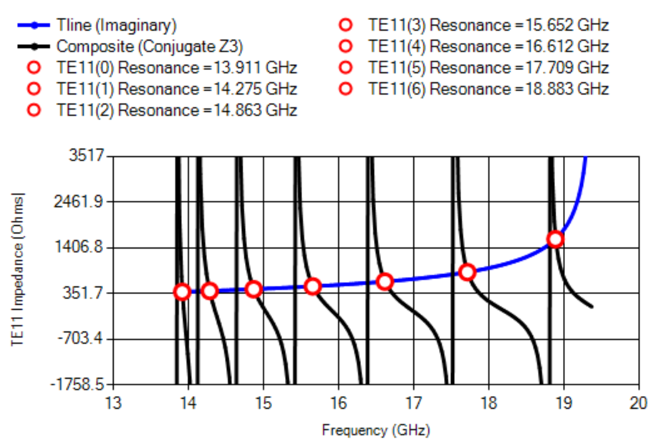


Figure #7a: Z_{0a} and Z_3 for bead length=1.000"

Figure #7b: Model and measured data for 1.000" bead



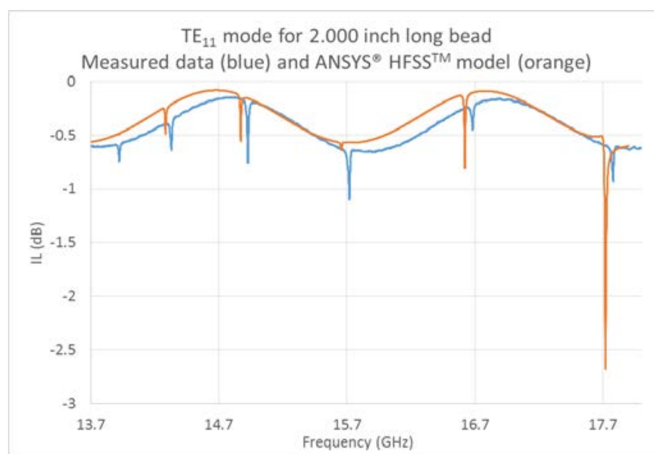


Figure #8a: Z_{0a} and Z_3 for bead length=2.000"

Figure #8b: Model and measured data for 2.000" bead

The very wide periodicity is due to the fact that the PTFE bead introduces a mismatch section within the airline. However, you can see that there is very good agreement between the theoretical calculations proposed by Gilmore[1] and the measured and modeled data for the TE_{11} resonances. Again, the simulated data was generated using ANSYS® HFSS™ software. Table #2 has a summary of the TE_{11} mode resonances for the various bead lengths.

Table #2: Effective TE_{11} mode frequencies for various bead lengths

Length (inches)	Resonance number	Measured TE11 (GHz)	Calculated TE11 (GHz)	Modeled HFSS TE11 (GHz)
0.125	1	17.474	17.499	17.5
0.25	1	16.015	16.012	16.019
0.5	1	14.825	14.827	14.835
0.5	2	17.632	17.622	17.628
1	1	14.165	14.176	14.186
1	2	15.283	15.294	15.303
1	3	17	17.007	17.017
2	1	13.923	13.911	13.92
2	2	14.329	14.275	14.284
2	3	14.927	14.863	14.871
2	4	15.721	15.652	15.659
2	5	16.684	16.612	16.621
2	6	17.779	17.709	17.72

Now, to make things more complicated, when two connectors are mated together, there will now be two beads in close proximity. Figure #9 shows an illustration of how Gilmore[1] represents the interface of the beads, in terms of input impedance labels. (Note, the connector reference plane would typically be half-way between t_g).

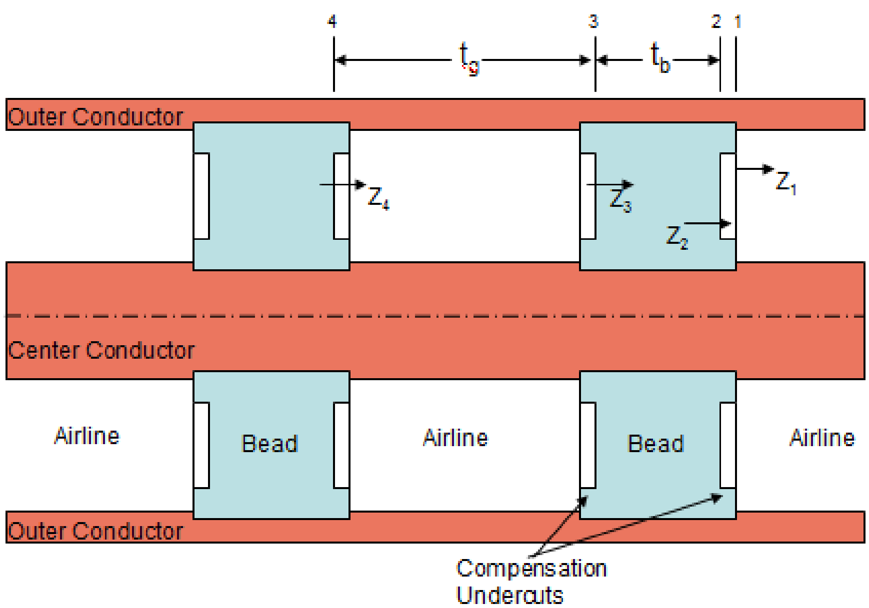


Figure #9: double bead illustration with the input impedance labels for each interface

To solve this configuration you use the same process as with the single bead. However, here you calculate Z_4 in terms of the gap transmission line and Z_3 as the termination. Since the input impedance, facing the opposite direction of Z_4 is just Z_3 (due to symmetry), you just need to see where Z_4 and the complex conjugate of Z_3 equal. The next page shows the formulas for Z_4.

$$Z_4 = Z_{0g} \frac{Z_3 + Z_{0g} \tanh\left(\gamma_g t_g\right)}{Z_{0g} + Z_3 \tanh\left(\gamma_g t_g\right)}$$

$$\text{where; } Z_{0g} = Z_{0a} \text{ ; } \gamma_g = \gamma_a = k_a \sqrt{1 - \left(\frac{f}{f_{ca}}\right)^2}$$

$$k_a = 2\pi \frac{f}{c_0} \quad \left(c_0 = \text{ speed of light in vacuum}\right)$$

If you do these calculations, you will see that the resonances that existed in the single bead split into two when two beads are brought in close proximity. Take the 0.250 inch bead example from before. It had a single resonance at about 16 GHz. When you separate the two beads by 0.500 inches, Figure #10 shows that the resonance splits into two parts, one at 15.896 GHz and one at 16.139 GHz.

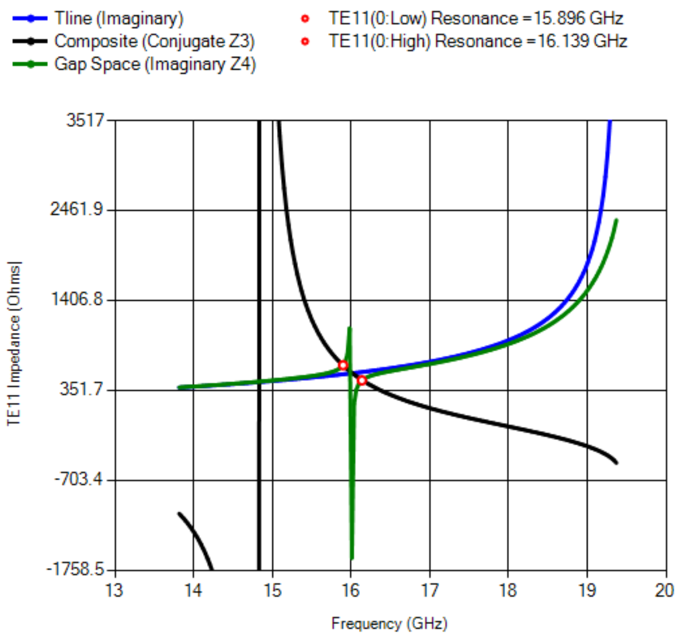


Figure #10: Two 0.250 inch beads, separated by 0.500 inches

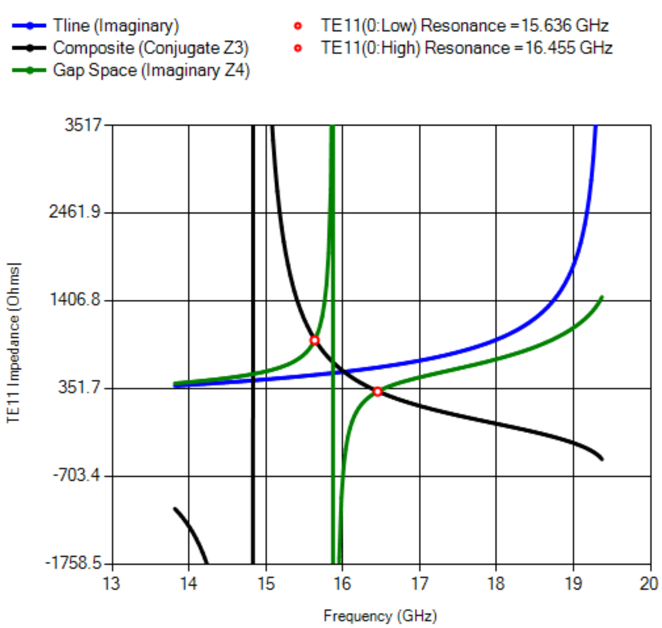


Figure #11: Two 0.250 inch beads, separated by 0.250 inches

Having the beads 0.250 inch from each reference plane (0.500 inches apart) is certainly within reason. However, if you wanted to see the extreme condition when they are moved closer, Figure #11 shows the beads separated by 0.250 inches, which yields two resonances at 15.344 GHz and 16.857 GHz.

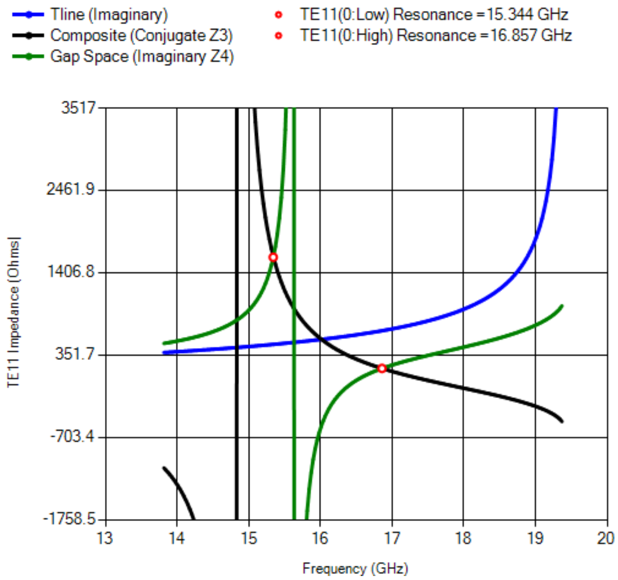


Figure #12: Two 0.250 inch beads, separated by 0.125 inches

Further, if you separate the beads by only 0.125 inches (shown in Figure #12), the two resonances separate to 15.344 GHz and 16.857 GHz.

Conclusions:

This paper describes two approaches to estimate the effective cut-off frequency of a coaxial connector, driven by the TE_{11} mode. The first is a theoretical calculation by Gilmore[1] that uses standard transmission line theory. The second uses a full field simulation tool, like ANSYS® HFSS™ software, to see how the electromagnetic fields actually propagate through the structure. The key to creating a model for field simulation, is to make sure you introduce a small non-concentric perturbation into the model in order to excite the higher order TE modes. It is also interesting to note what happens when you have two identical beads in close proximity, like one has with a mated pair of beaded connectors. Knowledge of these phenomena is critical to the designer trying to understand the effective cut-off, or max frequency of a connector design. This will allow them to optimize a design for the lowest possible loss in a coaxial cable assembly for a given maximum operating frequency.

References:

1) John F. Gilmore, "TE11-Mode Resonances in Precision Coaxial Connectors", *The General Radio Experimenter*, Vol. 40 Number 8, August 1966, pp. 10-13.
2) J. Dimitrios, "Exact Cutoff Frequencies of Precision Coax.", *Microwaves*, June 1965.